

# **Adomian Decomposition Method for Solving Boundary Value**

## **Problems for a Class of Partial Differential Equations**

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### **Abstract**

The Adomian decomposition method (ADM) is a method for solving nonlinear differential equations. The method was developed by George Adomian in the last twenty years. This method has many important applications especially in the fields of physics, chemistry, mechanics and other sciences. This study seeks to provide an illustration of how the method can be applied on the PDEs and on the Systems of PDEs. Moreover, this study offers an account of the modifications and development of this method since its inception till today, making it a basic building block that can serve any researcher in this field. In addition, the thesis discusses the Goursat's problem for linear and nonlinear hyperbolic equations of the second order. Also, it outlines the Goursat's problem for system of nonlinear hyperbolic equations of the second order as well as the Goursat's problem for linear and nonlinear hyperbolic equations of fourth order. The study also looks at the existence and uniqueness for the initial-Neumann boundary value problems for parabolic, hyperbolic equations and parabolic–hyperbolic equations in Hilbert space. The study describes a new iterative method similar to the Adomian decomposition method for the application of ADM in which initial conditions and boundary conditions (Dirichlet, Neumann and Mixed) are used to find a solution to the Initial- boundary value problems for parabolic, hyperbolic equations and parabolic–hyperbolic equations. Finally, the initial-boundary value problems for the parabolic, hyperbolic equations and parabolic–hyperbolic equations with integral boundary condition were solved by introducing a new function that transforms integral condition to a normal condition, then applying a new method taking into account all the initial and boundary conditions together in ADM with a new differential operator to arrive at solution. Moreover, the thesis has supported by various examples.

## Summary

The main goal of this thesis is to study a class of the Partial Differential Equations and a System of Partial Differential Equations by using Adomian decomposition method. This method was developed by Adomian (1980) and has gone through many modifications and development which allowed researchers to apply it on various problems.

This thesis consists of a general introduction and five chapters, and here below are details for the contents of these chapters.

**The first chapter** offers a general introduction to the Adomian decomposition method. This chapter also offers an illustration of how the method can be applied on the Partial Differential Equations and on the Systems of Partial Differential Equations. Moreover, it critically discusses the advantages and disadvantages of this method. This chapter concludes with a review of the modifications and development of this method since its inception till today along with examples.

**The second chapter** is devoted to the discussion of the Goursat's problem for linear and nonlinear hyperbolic equations of the second order. Also, we have studied the Goursat's problem for a system of nonlinear hyperbolic equations of the second order. In addition, the chapter concludes with a discussion of the Goursat's problem for linear and nonlinear hyperbolic equations of fourth order. We depend on the clue of transformation Goursat's problem with conditions on characteristics curves into equivalent classical problems, and then can be applied the Adomian decomposition method to obtain the exact solution.

**The third chapter** is concerned with the existence and uniqueness in Hilbert space for the initial-Neumann boundary value problems for parabolic, hyperbolic equations and parabolic–hyperbolic equations. Finding the solution was done by a series  $u(t, x) = \sum_{n=0}^{\infty} a_n(x) t^n$ . Then, a repetitive relationship was deduced  $a_n(x)$ . This method has the advantage of reducing the problem to an ordinary differential equation which can be handled very easily.

**The fourth chapter** describes a new iterative method similar to the Adomian decomposition method for the application of Adomian decomposition method in which initial conditions and boundary conditions (Dirichlet, Neumann and Mixed) to find a solution to the Initial-boundary value problems for parabolic, hyperbolic equations and parabolic–hyperbolic equations. This method is found to be very useful and gives accurate results. This is illustrated by examples.

**The fifth chapter** studies the initial-boundary value problems for the parabolic, hyperbolic equations and parabolic–hyperbolic equations with integral boundary condition that were solved by introducing a new function that transforms integral condition to a normal condition, then applying a new method taking into account all the initial and boundary conditions together in Adomian decomposition method with a new differential operator to obtain a solution.

This thesis includes lists of References, Figures, Tables and Terminology. This thesis ends with abstract and summary in English language, as well.

**The second, Third and fourth chapters were published in the journals listed below, respectively:**

Applied Mathematics

Journal of Applied Mathematics and Informatics(JAMI)

The International Journal of Nonlinear Science(IJNS)