Computers and Mathematics with Applications 62 (2011) 2364–2373

Contents lists available at SciVerse ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

A Chebyshev spectral method based on operational matrix for initial and boundary value problems of fractional order

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ARTICLE INFO

Article history:
Received 12 March 2011
Received in revised form 11 July 2011
Accepted 11 July 2011

Keywords:
Fractional differential equations
Spectral method
Shifted Chebyshev polynomials
Operational matrix
Caputo derivative

ABSTRACT

We are concerned with linear and nonlinear multi-term fractional differential equations (FDEs). The shifted Chebyshev operational matrix (COM) of fractional derivatives is derived and used together with spectral methods for solving FDEs. Our approach was based on the shifted Chebyshev tau and collocation methods. The proposed algorithms are applied to solve two types of FDEs, linear and nonlinear, subject to initial or boundary conditions, and the exact solutions are obtained for some tested problems. Numerical results with comparisons are given to confirm the reliability of the proposed method for some FDEs.

1. Introduction

It is known that many phenomena in several branches of science can be described very successfully by models using mathematical tools from fractional calculus. Methods of solutions of problems for fractional differential equations have been studied extensively by many researchers (see, e.g., [1–3] and the references given therein). The analytic results on the existence and uniqueness of solutions to the FDEs have been investigated by many authors; among them, [4,5]. In general, most of FDEs do not have exact analytic solutions, so approximation and numerical techniques must be used.

Finding accurate and efficient methods for solving FDEs has become an active research undertaking. There are several analytic methods such as the Adomian decomposition method [6,7], the homotopy-perturbation method [8], the variational iteration method [9] and the homotopy analysis method [10]. From the numerical point of view, Diethelm et al. [11] presented the predictor–corrector method for numerical solutions of FDEs. In [12], the authors have proposed an approximate method for the numerical solution of a class of FDEs which are expressed in terms of Caputo type fractional derivatives. In fact, the method presented in [12] takes advantage of FDEs converting into Volterra-integral equations. In [7], analytical and numerical methods are used to solve a multi-term nonlinear fractional differential equation. Furthermore, the generalization of the Legendre operational matrix to the fractional calculus has been studied in [13].

The main advantage of spectral methods lies in their accuracy for a given number of unknowns. For smooth problems in simple geometries, they offer exponential rates of convergence/spectral accuracy (see, e.g., [14–17]). In the present paper, we extend the application of spectral methods with generalization of Chebyshev operational matrix (COM) to the fractional calculus for developing direct solution techniques for solution of linear multi-term FDEs.

Doha et al. [18] proposed an efficient spectral tau and collocation methods based on the Chebyshev polynomials for solving multi-term linear and nonlinear fractional differential equations subject to nonhomogeneous initial conditions.

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doi:10.1016/j.camwa.2011.07.024