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C-admissibility and analytic C-semigroups

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ABSTRACT

We introduce the notion of C-admissible subspaces and obtain various conditions of C-admissibility, generalizing well known results of Vu and Schuler. Moreover, we show the uniqueness of solutions for the operator equation $AX - XB = CD$ with A generating an analytic C-semigroup which generalize results of Vu.

1. Introduction

We introduce the notion of C-admissible subspaces which is a generalization of the notion of admissible subspaces, and we extend to this situation the results of Vu and Schuler [1].

In [2], Vu proved the following theorem:

Theorem 1.1. *Let A be a generator of an analytic semigroup in a Banach space E. Assume that B is a closed linear operator in a Banach space F such that $\overline{\Sigma_{\omega,\theta}} \subset \rho(B)$ and $\|\lambda(B-\lambda)^{-1}\|$ is uniformly bounded when λ belongs to the sector $\Sigma_{\omega,\theta}$ (where $\Sigma_{\omega,\theta} = \{\lambda \in \mathbb{C} : |\arg(\omega-\lambda)| < \theta\} \cup \{\omega\}$ and $\sup_{\lambda\in\mathbb{C}\setminus\Sigma_{\omega,\theta}} \|\lambda(A-\lambda)^{-1}\| < \infty$). Then, the operator equation*

$$AX - XB = D$$

has a unique solution which is expressed by

$$X = \frac{1}{2\pi i}\int_\Gamma (A-\lambda)^{-1} D (B-\lambda)^{-1} d\lambda.$$

In Section 4, we generalize such theorem to the case where A is a generator of an analytic C-semigroup.

2. C-admissibility

In this section, we introduce the notion of C-admissible subspaces which is a generalization of the notion of admissible subspaces, and we extend to this situation the results of Vu [2] and Schuler and Vu [1].

Consider the differential equation

$$u'(t) = Au(t) + f(t), \quad t \in \mathbb{R} \qquad (*)$$

where A is a closed linear operator on a Banach space E and f is a continuous function from $\mathbb{R}$ to E.

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