Available online at www.sciencedirect.com
ScienceDirect
Nonlinear Analysis 68 (2008) 2181–2193
Nonlinear Analysis
www.elsevier.com/locate/na

Fixed points for generalized contractions and applications to control theory

Hemant Kumar Pathak[a], Naseer Shahzad[b,*]

[a] School of Studies in Mathematics, Pt. Ravishankar Shukla University, Raipur (C.G.), 492001, India
[b] Department of Mathematics, King Abdul Aziz University, P.O. Box 80203, Jeddah 21589, Saudi Arabia

Received 4 January 2007; accepted 23 January 2007

Abstract

The concepts of "weak/strong topological contraction" and a generalization of Banach contraction mappings called "p-contraction" are introduced and used to prove fixed point theorems for self-mappings from a topological/metric space into itself satisfying topological contraction/metric p-contraction, respectively. Certain non-linear integral equations defined on $C[a, b]$ satisfying generalized Lipschitzian conditions can easily be solved by applying these theorems. In the sequel, we shall study the possibility of optimally controlling the solution of the ordinary differential equation via dynamic programming.

MSC: 54H25; 47H10

Keywords: p-contraction; Fixed points; Dynamic programming

1. Introduction

In this note, we provide one more affirmative answer to an open question of Rhoades [18, p. 242] of whether or not there exists a contractive definition which is strong enough to generate a fixed point but which does not force the map to be continuous at the fixed point. It may be observed that in most of the fixed point theorems in the literature either continuity is explicitly assumed or, as shown by Rhoades [18] and Hicks and Rhoades [9], the contractive definitions themselves imply continuity at the fixed point. However, an affirmative answer was given by Pathak et al. in [16, Example 2.1].

The applications of fixed point theorems are very important in diverse disciplines of mathematics, statistics, engineering and economics in dealing with problems arising in: approximation theory, potential theory, game theory, mathematical economics, theory of differential equations, theory of integral equations, etc.

2. Preliminaries and definitions

Let (X, d) be a metric space and let T be a mapping of X into itself. An orbit of T at a point x of X is the set

$$0(x, T) := \{x, Tx, \dots, T^n x, \dots\}.$$

* Corresponding author.
E-mail addresses: hkpathak@sify.com (H.K. Pathak), nshahzad@kau.edu.sa, naseer_shahzad@hotmail.com (N. Shahzad).

0362-546X/$ - see front matter
doi:10.1016/j.na.2007.01.041